A possible enhancement of nuclear fission in scattering with low energy charged particles

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New mechanisms for tunneling enhancement

- Tunneling in scattering with multiple degrees of freedom [G. F. Bonini, A. G. Cohen, C. Rebbi and V. A. Rubakov, 1999]
- Alpha-decay induced by collision with low energy protons [B. Ivlev and V. G., 2004]
- Influence of intrinsic structure of composed particles on the tunneling [C. A. Bertulani, V. V. Flambaum and V. G. Zelevinsky, 2007]
- ....
- Could be important for sub-barrier and close-to-barrier nuclear fission.

Semi-classical approach

$$W \sim \exp(-A(E))$$

$$A(E) = \frac{2}{\hbar}\int dx\sqrt{2m[V(x)-E]}$$

$$A \sim \frac{V}{\hbar\omega} \gg 1$$

${}^{235}_{92}U + p \to {}^{231}_{90}Th + \alpha + p$

$E = 4.678 MeV$

Numerical 2-dim: $\Delta E = 0 MeV$ for $\varepsilon_0 \approx 1.85 MeV$ when $\phi \approx 30^\circ$

"Resonance":

$\varepsilon_{\mathrm{R}} \approx 0.25 MeV$ with $\Delta E = -1.15 MeV$ and $\phi \approx 11^\circ$.

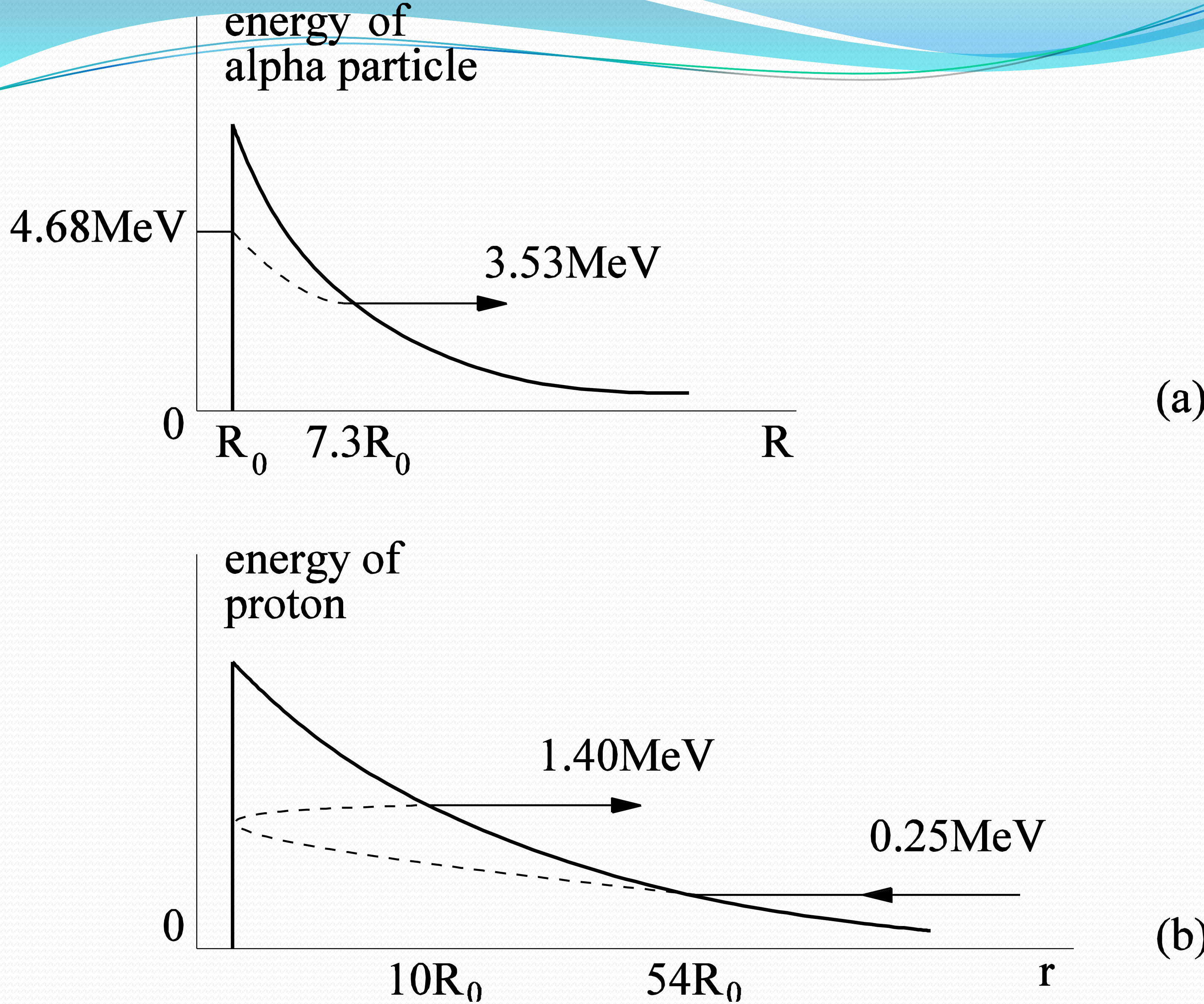
Then

$E - |\Delta E| \approx 3.53 MeV$ and $\varepsilon + |\Delta E| \approx 1.40 MeV$

The shape of the pick

$$\sim \exp\left(-\frac{\varepsilon - \varepsilon_R}{\Delta\varepsilon}\right)$$ with $\Delta\varepsilon \approx 3 KeV$

B. Ivlev & V.G. (2004)


energy of
alpha particle
4.68MeV
3.53MeV
0
R_0
$7.3R_0$
R
(a)
energy of
proton
1.40MeV
0.25MeV
0
$10R_0$
$54R_0$
r
(b)

Fission

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = (H_0 + V_t)\Psi(x,t)$$

$$V_t = \frac{eZ}{R(t)} \qquad \rightarrow \qquad V_\xi \quad \text{with} \;\; r(\xi) = a(\sigma \cosh \xi + 1)$$

$$t = \gamma(\sigma \sinh \xi + \xi); \qquad \gamma = \sqrt{\frac{m_p a^3}{eZ}}; \qquad a = \frac{eZ}{2E_p}$$

$$\sigma = \sqrt{1 + \frac{2E_p \hbar^2 l(l+1)}{m_p (eZ)^2}}$$

Proton potential

$$V_t = \frac{A_1}{|\vec{r}_1 + \vec{R}(t)|} + \frac{A_2}{|\vec{r}_2 + \vec{R}(t)|}; \quad |R| \gg |r_1|, |r_2|$$

$$\Rightarrow$$

$$V_t \simeq \frac{(A_1 + A_2)}{R} + \frac{(A_2 r_2 - A_1 r_1)}{R^2}\cos\varphi + \frac{(A_1 + A_2)}{2R^3}(r_1^2 + r_2^2) + \ldots$$

Toy model

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{2}\omega^2(t)x^2\psi$$

where

$$\omega^2(t)=\omega_0^2+\frac{eZ}{4a^3m_p(\sigma\cosh\xi+1)^3}=\omega_0^2(1+\eta/(\sigma\cosh\xi+1)^3)$$

$$\eta=\frac{2}{(\alpha Z)^2}\left(\frac{E_p}{\omega_0\hbar}\right)^2\left(\frac{E_p}{m_pc^2}\right)\sim10^{-4}-10^{-5}\quad\text{for}\quad E_p\sim1MeV$$

Quantum parametric resonance

$\omega^2(t) = \omega_0^2(1 + 2\varepsilon \sin((2+\delta)\omega_0 t))$ at $|\varepsilon|,|\delta| \ll 1$

Fourier Series:

$t \in [-\pi/\omega_0, \pi/\omega_0]$ then $\dfrac{\pi}{\omega_0 \gamma} \geq |\sigma \sinh \xi + \xi|$

$$\omega_0 \gamma = (\alpha Z)\left(\frac{\omega_0 \hbar}{E_p}\right)\sqrt{\frac{m_p c^2}{E_p}} \sim 10 - 10^2 \gg 1$$

then $\xi \gg 1$

- (L. P. Pitaevsky, A. M. Perelomov, Ya. B. Zeldovich)

QPR-2

$$\varepsilon \sim \frac{1}{(\alpha Z)^2}\left(\frac{E_p}{\omega_0 \hbar}\right)^2\left(\frac{E_p}{m_p c^2}\right) \sim 2\cdot(10^{-3}-10^{-4}) \text{ for } E_p \sim 1MeV$$

$$\Gamma \sim \varepsilon\omega_0 \sim 0.2KeV - 2KeV$$

L. D. Landau, E. M. Lifshitz

QPR-barrier penetration

For $|\varepsilon| \sim |\delta| \ll 1$

$$T \sim \frac{\varepsilon^2 \omega_B^2 t^2 / 4}{1 + \varepsilon^2 \omega_B^2 t^2 / 4}$$

$t \sim D / v_p = (eZ / E_p) / (E_p / 2m_p)$ and $\omega_B = E_B / \hbar$

$\Rightarrow$

$$\omega_B t \sim 2(\alpha Z) \frac{E_B (m_p c^2)}{E_p^2}$$

Then:

$\varepsilon \omega_B t \sim \frac{2}{(\alpha Z)} \left(\frac{E_p}{E_B} \right) \sim \frac{1}{2}$--1 and $T \sim 0.1 - 0.5$

- (L. P. Pitaevsky, A. M. Perelomov, Ya. B. Zeldovich)

Conclusion

- The possible manifestation of parametric resonance enhancement of fission could be an observation of narrow $(0.2KeV - 2KeV)$ but very strong resonances in induced by collision with low energy protons.