

MDDC - 905

UNITED STATES ATOMIC ENERGY COMMISSION

A GENERAL METHOD FOR DETERMINING COINCIDENCE
CORRECTIONS OF COUNTING INSTRUMENTS

by

T. P. Kohman

LIBRARY OF ENERGY
SCIENCE & TECHNOLOGY PROJECT
TECHNICAL INFORMATION SERVICE

W4849

OCT 18 1948

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

Hanford Engineering Works

DTIC QUALITY INSPECTED 4

This document consists of 18 pages.
Date of Manuscript: June 13, 1945
Date Declassified: April 14, 1947

Its issuance does not constitute authority
for declassification of classified copies
of the same or similar content and title
and by the same author.

Technical Information Division, Oak Ridge Directed Operations
AEC, Oak Ridge, Tenn., 9-24-48-1500

Printed in U.S.A.
PRICE 10 CENTS

19961216 071

A GENERAL METHOD FOR DETERMINING COINCIDENCE
CORRECTION OF COUNTING INSTRUMENTS

By T. P. Kohman

ABSTRACT

The method herein described for determining and applying coincidence corrections in counters is an extension of the general method of paired sources. Several pairs of radioactive sources are used, and for each pair, counts are taken with the instrument on the sources separately and together. A general relationship between the recorded counting rate R and the "true" rate N is assumed, being the first few terms of an infinite power series:

$$N = R + \tau R^2 + \nu R^3 + \phi R^4 + \dots$$

where τ is the resolving time of the counter and ν, ϕ, \dots are independent parameters. By a least-squares method, the values of the parameters for the instrument in question are determined which most adequately correlate the experimental data. Subsequent measurements made with the instrument can then be corrected by means of this equation.

A few examples of the use of this method are presented. They indicate that the method is a satisfactory solution of the problem and should be of considerable value.

INTRODUCTION

All instruments which count random events, such as those encountered in the measurement of radioactivity, have finite resolving powers, and hence are unable to separately distinguish and record events occurring very close together in time. As a consequence some counts are missed, the fraction lost increasing as the counting rate increases. This results in a nonlinear variation of the response of the instrument to the intensity of the source of events. The difference between the recorded rate and the "true" rate (that is, the rate which would be observed if the instrument could count all events to which it is normally sensitive) is called the coincidence loss or correction.

In practice coincidence losses are often appreciable, and therefore, in order to obtain accurate measurements in radioactivity with counting instruments, it is necessary to correct the recorded counting rates for this effect. Moreover, the useful upper limit of a counter is greatly extended by a knowledge of its coincidence corrections. Coincidence loss measurements also supply valuable information about the mode of operation of counters, and provide a useful guide in improving instruments so as to decrease their losses.

The problem has received attention for a number of years, and several methods of determining and applying coincidence corrections are described in the literature. However, all of these methods are either inconvenient, not generally applicable, or not sufficiently accurate. Recent experience at the Hanford Engineer Works and other Metallurgical Project Sites has indicated the importance of the problem and the inadequacy of the previously described methods. This report describes a new method which is capable of any desired accuracy and which is almost completely general in its applicability.

THEORY OF THE METHOD

Notation

To facilitate the discussion, the notation to be used is presented herewith:

R = recorded rate.

N = true or corrected rate (including background).

b = background rate.

$N' = N - b$ = corrected rate due to source alone.

t = time of count.

Subscripts A and B refer to individual members of a pair of sources.

Subscript C refers to the combined sources of the pair.

τ = resolving time of counter.

ν, ϕ, \dots = supplementary parameters.

Methods Involving the Use of Paired Sources

Reviews of methods of determining coincidence corrections have been given by Beers¹ and Kohman.² The use of paired sources, first proposed by Moon,³ has been frequently employed.^{4,5,1,2,8,7} If each of two radioactive sources of approximately equal strength is measured separately, and then the two are measured together, taking care to keep geometrical factors constant throughout, the combined counting rate (corrected for background) will be less than the sum of the individual rates (corrected for background), since the fractional counting loss is greater at the higher counting rate. If a simple mathematical relationship between recorded and true rates involving but a single parameter applies for the instrument, a single set of measurements on one pair of sources can be used to evaluate the parameter.^{4,1,2,8,8,7} However, it has been shown^{2,8} that the simple one-parameter relationships previously proposed do not adequately express the behavior of real counters.

In order to avoid this difficulty, a completely general method which involves no assumed mathematical relationship between true and recorded rates was developed.² This method employs a number of paired sources of different intensities covering the range of the instrument, and the data are analyzed by a graphical method of successive approximations which gives a curve of fractional correction versus recorded rate. Unfortunately, this method is tedious, time consuming, and possibly subject to personal factors. A completely mathematical analysis of the data would be a considerable improvement. A prerequisite of this is a mathematical relationship between true and recorded rates.

Single-parameter relationships between N and R which have been proposed are these:

a) Johnson and Street,⁹ Ruark and Brammer:¹⁰

$$N = R + \tau NR \quad (1)$$

b) Volz,¹¹ Schiff:¹²

$$N = Re^{\tau N} \quad (2)$$

c) An approximation of (b) due to Hull:⁴

$$N = R + \tau R^2 + 1/2\tau^2 R^3 \quad (3)$$

d) An approximation of (a)^{6,2,7}:

$$N = R + \tau R^2 \quad (4)$$

For each of these relationships, methods have been developed for evaluating the parameter τ from a set of paired source measurements. The derivations of equations 5 to 9 proceed from the fact that $N'_A + N'_B = N'_C$.

a) Beers:¹

$$\tau = \frac{R_A + R_B - R_C - b}{2(R_A - b)(R_B - b)} \frac{[(R_A + R_B - R_C - b)(R_C - b)/(R_A - b)(R_B - b)]^2}{8(R_C - b)} \quad (5)$$

Kohman:⁶

$$\tau = \frac{1}{R_C} \left(1 - \sqrt{1 - \frac{R_C}{R_A R_B} (R_A + R_B - R_C - b)} \right) \quad (6)$$

Equation 5 is more exact at low counting rates, and equation 6 at high rates.

b) Crawford:⁶

$$N_C = (R_A + R_B)^2 / R_C \quad (7)$$

τ is then obtained from N_C , R_C , and equation 2. For equation 7 to be applicable, the background must be small enough to be neglected, and R_A and R_B must be very nearly equal.

c) Hull:⁴

$$1/2 (R_A^3 + R_B^3 - R_C^3)\tau^2 + (R_A^2 + R_B^2 - R_C^2)\tau + R_A + R_B - R_C - b = 0 \quad (8)$$

τ is obtained by solution of this equation by the quadratic formula.

d) Jarrett;⁸ Kohman;^{2,3} Metcalf and Hennessee:⁷

$$\tau = (R_A + R_B - R_C - b)/(R_C^2 - R_A^2 - R_B^2) \quad (9)$$

Inspection of equations 1, 2, 3, and 4 reveals that they are all nearly equivalent for low counting rates, where the difference between N and R is very small ($\sim 1\%$). Moreover, any one will adequately represent the behavior of actual counters in the region of small losses. Equation 4 means that the fractional correction is proportional to R , and because of the simplicity of this equation and its corollary 9, these equations have received extensive use in rough evaluations and applications of coincidence corrections.^{6,13,8,7} In general, however, no single-parameter equation, theoretical or empirical, can adequately relate N and R when the losses are large. As an illustration, a semi-theoretical equation which expressed the observed behavior of a Geiger-Mueller counter over a wide range of counting rates contained three parameters (equation 2). However, that equation is not suitable for practical application, nor is it applicable to other types of instruments.

In theory, the relationship between two dependent variables such as N and R can be expressed by an infinite power series:

$$N = c_0 + c_1 R + c_2 R^2 + c_3 R^3 + c_4 R^4 + \dots \quad (10)$$

If we set $c_0 = 0$

$$c_1 = 1$$

$$c_2 = \tau$$

equation 10 becomes equivalent to equations 1, 2, 3, and 4 for small values of R. Hence, we are justified in rewriting the series as follows:

$$N = R + \tau R^2 + \nu R^3 + \phi R^4 + \dots \quad (11)$$

For practical purposes, the number of terms of equation 11 to be retained will be determined by the behavior of the instrument and precision required.

As previously mentioned, when only two terms of equation 11 are used, giving equation 4, the parameter can be evaluated from measurements on a single pair of sources by means of equation 9. It has been pointed out by Jarrett⁶ that one might similarly use the first three terms of equation 11 and evaluate the two parameters from measurements on two pairs of sources. One obtains two equations analogous to equation 9 but involving the two unknown parameters, and their simultaneous solution yields the values of the latter. Similarly, this method can be extended to any number of terms of equation 11 by employing as many pairs of sources as there are parameters.

The Least-squares Evaluation of Paired Source Data

The use of equations such as equation 9, or its higher analogues according to the method of Jarrett, has a serious disadvantage in that only a limited number of measurements can be admitted into the evaluation of the parameters. It is generally desirable to employ a greater number of measurements in obtaining the final result, so that an error in one measurement will not seriously effect the values of the parameters (as would be the case when the minimum number of measurements is used), and so that the entire counting range of the instrument can be represented in the data. The method of least squares provides a means of doing this.

Consider a pair of sources which have been counted separately and together. Let us define the quantity δ , which we will call the residual, thus:

$$\delta = N'_A + N'_B - N'_C$$

If the N values were the true values, as would be the case if the correction of the R values were perfect, then δ would equal zero (except for statistical fluctuations due to the random distribution of counts). Actually, when the N values are obtained by an equation such as will be used in practice, the corrections will not be perfect, and $\delta \neq 0$. δ is the algebraic sum of the errors in the corrections to R_A , R_B , and R_C . It may be considered as a measure of the error in the correction to R_C since this is the largest correction, and hence, is expected to have the largest error. Similarly, we may define the fractional residual $\epsilon = \delta/R_C$, which may be considered a measure of the fractional error in the correction to R_C .

If we have a number of such paired source measurements, the best values of the parameters in equation 11 are those which make $\Sigma \epsilon^2$ a minimum:

$$\begin{aligned} \frac{\partial}{\partial \tau} \sum \epsilon^2 &= 0 \\ \frac{\partial}{\partial \nu} \sum \epsilon^2 &= 0 \\ \frac{\partial}{\partial \phi} \sum \epsilon^2 &= 0, \text{ etc.} \end{aligned}$$

One-parameter Case

For the case where only two terms of equation 11 are retained, giving equation 4, the evaluation of the parameter τ by the least-squares method proceeds as follows:

$$\begin{aligned}\epsilon &= \frac{\delta}{R_C} = \frac{N'_A + N'_B - N'_C}{R_C} \\ &= \frac{N_A + N_B - N_C - b}{R_C} \\ &= \frac{R_A + R_B - R_C - b + \tau(R_A^2 + R_B^2 - R_C^2 - b)}{R_C} \\ &= \frac{D + \tau E}{R_C}\end{aligned}$$

where $D = R_A + R_B - R_C - b$

$$E = R_A^2 + R_B^2 - R_C^2$$

$$\begin{aligned}\sum \epsilon^2 &= \sum \frac{1}{R_C^2} (D + \tau E)^2 \\ &= \sum \frac{1}{R_C^2} (D^2 + 2\tau DE + \tau^2 E^2) \\ &= \sum \frac{D^2}{R_C^2} + 2\tau \sum \frac{DE}{R_C^2} + \tau^2 \sum \frac{E^2}{R_C^2} \\ &= G + 2\tau H + \tau^2 L\end{aligned}$$

where

$$G = \sum \frac{D^2}{R_C^2}$$

$$H = \sum \frac{DE}{R_C^2}$$

$$L = \sum \frac{E^2}{R_C^2}$$

The condition for best fit is

$$\frac{d}{d\tau} \sum \epsilon^2 = 0 = 2H + 2\tau L$$

whence:

$$\tau = -(H/L)$$

(12)

Two-parameter Case

For the case where three terms of equation 11 are used:

$$N = R + \tau R^2 + \nu R^3 \quad (13)$$

the least-squares evaluation of the two parameters follows:

$$\begin{aligned} \epsilon &= \frac{R_A + R_B - R_C - b + \tau(R_A^2 + R_B^2 - R_C^2) + \nu(R_A^3 + R_B^3 - R_C^3)}{R_C} \\ &= \frac{D + \tau E + \nu F}{R_C} \end{aligned}$$

where $D = R_A + R_B - R_C - b$

$$E = R_A^2 + R_B^2 - R_C^2$$

$$F = R_A^3 + R_B^3 - R_C^3$$

$$\begin{aligned} \sum \epsilon^2 &= \sum \frac{1}{R_C^2} (D + \tau E + \nu F)^2 \\ &= \sum \frac{1}{R_C^2} (D^2 + 2\tau DE + 2\nu DF + 2\tau\nu EF + \tau^2 E^2 + \nu^2 F^2) \\ &= \sum \frac{D^2}{R_C^2} + 2\tau \sum \frac{DE}{R_C^2} + 2\nu \sum \frac{DF}{R_C^2} + 2\tau\nu \sum \frac{EF}{R_C^2} + \tau^2 \sum \frac{E^2}{R_C^2} + \nu^2 \sum \frac{F^2}{R_C^2} \\ &= G + 2\tau H + 2\nu J + 2\tau\nu K + \tau^2 L + \nu^2 M \end{aligned}$$

where $G = \sum \frac{D^2}{R_C^2}$

$$H = \sum \frac{DE}{R_C^2}$$

$$J = \sum \frac{DF}{R_C^2}$$

$$K = \sum \frac{EF}{R_C^2}$$

$$L = \sum \frac{E^2}{R_C^2}$$

$$M = \sum \frac{F^2}{R_C^2}$$

The condition for best fit is:

$$\frac{\partial}{\partial \tau} \sum \epsilon^2 = 0 = 2H + 2vK + 2\tau L$$

$$\frac{\partial}{\partial v} \sum \epsilon^2 = 0 = 2J + 2\tau K + 2vM$$

Solving these simultaneous equation yields:

$$\tau = (JK - HM)/(LM - K^2)$$

$$v = (HK - JL)/(LM - K^2) \quad (14)$$

Three-parameter Case

The derivation of the method for the case where three parameters are used

$$N = R + \tau R^2 + v R^3 + \phi R^4 \quad (15)$$

proceeds similarly. One obtains three simultaneous equations in three unknowns, whose solution yields the desired values. The solution is straightforward, but the computation is somewhat longer than in the two-parameter case, especially since the solution of simultaneous equations containing more than two unknowns is somewhat involved. Consequently, the two-parameter case is probably the most satisfactory for routine practical use. Only if one needs high accuracy and the experimental measurements have been made with a compatible degree of precision, or if the fractional corrections are quite large (~50%) should it be necessary to invoke additional parameters. Obviously the method can be extended to any number of parameters.

Significance of the Parameters

It is customary to consider the coincidence losses of a counter in terms of quantities called the resolving time, the insensitive time, and the dead time. The resolving time is defined as the minimum interval between two events which can both be registered by the instrument. The insensitive time is the interval following a recorded event during which the instrument is incapable of recording another event. The dead time is a quantity peculiar to Geiger-Mueller counters, and is the interval following a discharge during which the tube is incapable of producing an electron avalanche.

The insensitive time is not a property of the instrument, but of a particular event recorded by the instrument. With practically all types of counters, this quantity is not a constant, but may be different for different events. For example, the insensitive time of a Geiger-Mueller counter discharge may be shortened by the occurrence of another discharge a short time previously,^{14,2,6} or it may be lengthened by the occurrence of an unrecorded discharge near the end of the interval. The insensitive time of a pulse in an alpha particle counter may depend on the length and position of the track of the particle. If the insensitive time had a constant value, the resolving time would have this same value. But, in view of the variability of insensitive times, the simple definition for resolving time given has no clear meaning. Nevertheless, it is desirable to retain the idea of the resolving time as a characteristic of the instrument, in the sense that the insensitive time is a characteristic of an individual event.

Equations 1 and 2 are derived theoretically from certain physical assumptions, among which is that of a constant insensitive time or resolving time, τ . The parameter has the same meaning in other approximate equations, such as equations 3 and 4. Since the empirical equation 11 becomes essentially equivalent to these equations at very low counting rates, we may call τ in the latter also the resolving time. Physically, it is the limiting average value of the insensitive time as the counting rate approaches zero. With Geiger-Mueller counters, the insensitive time is constant at very low

rates, but with proportional counters or counting ionization chambers, the insensitive time varies, so the average is specified. Mathematically, the resolving time may be defined as

$$\tau = \left[\frac{d(N/R)}{dR} \right]_{R=0} = - \left[\frac{d(R/N)}{dN} \right]_{N=0}$$

The parameters ν, ϕ, \dots of equation 11 have no simple physical significance, but are merely constants employed to correlate the experimental observations.

The parameter τ , being the resolving time, has the dimension time and is always positive. The parameters ν, ϕ, \dots have the dimensions $\text{time}^2, \text{time}^3, \dots$, and may have either sign.

The discussion thus far has been confined to counting instruments. The method is equally applicable to some other types of measuring instruments which do not count. All radiation measuring instruments are expected to deviate increasingly from linearly as the radiation intensity increases. Examples to which the method can be applied are Lauritsen quartz-fiber electroscopes, high pressure gamma-ray ionization chambers, beta-ray ionization chamber, DC amplifier assemblies, photocells, etc. In such cases, the notion of a resolving time no longer applies, and it is more logical to rewrite equation 11 as

$$I = M + aM^2 + bM^3 + cM^4 + \dots \quad (16)$$

where M and I are the measured and true intensities respectively.

Once the parameters have been evaluated for a particular instrument by the method described, subsequent measurements made with the instrument can be corrected by the use of the corresponding equations 4, 13, 15 The coincidence correction should be added before subtracting the background, especially if both are large. To facilitate the application of corrections, one may employ a graph or table showing the correction as a function of the recorded rate.

APPLICATION OF THE METHOD

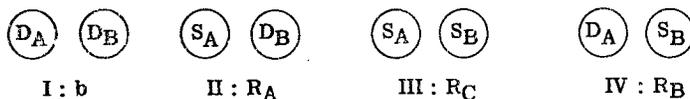
Procedure for Taking Data

The experimental part of the determination of coincidence corrections of an instrument consists of taking a series of sets of paired source measurements. Each set is comprised of four measurements involving two approximately equal sources: R_A, R_B, R_C and b . The series should contain three or more sets, with counting rates covering the range of the instrument for which the corrections are desired.

The procedure for taking a set of measurements is discussed. Sources and counting conditions are chosen which will give the desired counting rates. With many instruments the counting rate can be adjusted by varying the distance between source and detector, or by inserting suitable absorbers. In order to eliminate possible changes due to variations in geometry, a dummy source should occupy the place of each active source when the latter is not in position. The order of measurements is:

- 1) Place both dummies in position. Measure b .
- 2) Replace dummy A by active source A. Measure R_A .
- 3) Replace dummy B by source B, with care not to disturb the position of source A. Measure R_C .
- 4) Replace source A by dummy A, with care not to disturb the position of source B. Measure R_B .

The sequence is illustrated in the figure below, where S_A and S_B are the active sources and D_A and D_B the dummies:



It is generally desirable to count R_A , R_B , and R_C for equal periods of time. In this manner, one can obtain the greatest statistical accuracy for a given total number of counts.¹ This is desirable with instruments such as Geiger-Mueller counters whose lifetime is a function of usage. The greatest statistical accuracy for a given total expenditure of time is obtained when:

$$t_A : t_B : t_C : t_b = \sqrt{R_A} : \sqrt{R_B} : \sqrt{R_C} : \sqrt{b} \approx 1 : 1 : \sqrt{2} : \sqrt{b/R_A}$$

However, the loss in accuracy by making $t_A = t_B = t_C$ is slight, so even for instruments which are not damaged by usage, the convenience of using equal counting periods is sufficient to justify it. It is also desirable to make t_b greater than indicated by this ratio, since a more exact value of the background is a useful indication of whether the instrument is functioning properly.

The counting time necessary for each set depends on the accuracy required in the corrections, and may be estimated as discussed. Consider an instrument for which equation 4 holds, and τ has a value of approximately 10^{-5} minute. We wish to determine τ from a single set of paired source measurements using equation 9, the counting rates being approximately 1000, 1000, and 2000 per minute. We require an accuracy (fractional probable error) of 0.5% in corrected measurements at 2000 c/m. The coincidence correction at this rate is approximately 40 c/m, and this quantity must be known with an accuracy of 10 c/m, or 25% of its value. This means that τ must be known with an accuracy of 25%, which in turn means that $D (= R_A + R_B - R_C - b)$ must also have an accuracy of 25%. Since $D \approx 20$ c/m, its probable error must be 5 c/m. The probable error in D is equal to

$$0.6745 \sqrt{\frac{R_A}{t_A} + \frac{R_B}{t_B} + \frac{R_C}{t_C} + \frac{b}{t_b}} \approx 0.6745 \sqrt{\frac{2R_C}{t}}$$

when the counting times are equal. From this it follows that $t = 73$ minutes. The general formula for the counting interval is:

$$t \approx \frac{8K^2}{P^2 R_C} = \frac{3.64}{P^2 R_C} \quad (17)$$

where $t = t_A = t_B = t_C =$ required counting interval

$P =$ fractional probable error required at $R = R_C$

$K = 0.6745 =$ factor used in computing probable errors.

(If it is desired to express P in terms of some other measure of deviation than the probable error, the corresponding value of K must be used instead.)

It is to be noted that the time requirement is independent of the resolving time. From equation 17, Table 1 has been prepared.

In cases where equation 4 does not hold, and the method described herein is to be used instead, we may imagine each set of paired source measurements as establishing a point on the correction curve. Hence, the same considerations apply, and equation 17 and Table 1 can be used as a general guide in paired source measurements. Thus, when a series of such measurements is made, each set should contain approximately the same total number of counts.

Table 1. Length of counting interval t in minutes required for a given fractional probable error P for various combined source counting rates R_C in counts/minute.

$R_C \backslash P$	5%	2%	1%	0.5%	0.2%	0.1%
1,000	1.5	9.1	36	146	910	3640
2,000	0.7	4.6	18	73	455	1820
5,000	0.3	1.8	7.3	29	182	728
10,000	0.15	0.9	3.6	15	91	364
20,000	0.07	0.5	1.8	7.3	46	182
50,000	0.03	0.2	0.7	2.9	18	73
100,000	0.015	0.09	0.4	1.5	9.1	36
200,000	0.007	0.05	0.2	0.7	4.6	18
500,000	0.003	0.02	0.07	0.3	1.8	7.3
1,000,000	0.001	0.01	0.04	0.15	0.9	3.6

Table 2. Paired source measurements on Geiger-Mueller counter. Argon alcohol-filled, bell type, mica window tube with self-quenching circuit and Offner scaler.

No.	R_A (c/m)	R_B (c/m)	R_C (c/m)	b	τ_1 (min)	δ (c/m)	ϵ (%)
I	2334	2485	4750	30	3.6×10^{-6}	- 6	-0.13
II	3077	3155	6133	28	3.9×10^{-6}	- 4	-0.07
III	3089	3116	6134	28	2.3×10^{-6}	-33	-0.54
IV	3727	3696	7293	33	3.8×10^{-6}	- 9	-0.12
V	3779	3692	7324	29	4.6×10^{-6}	+12	+0.16
VI	5452	5356	10505	33	5.2×10^{-6}	+56	+ 0.53
VII	5334	5411	10508	34	3.9×10^{-6}	-14	-0.13

Result of computation: $\tau = 4.12 \times 10^{-6}$ min, and $N = R + \tau R^2$.

Table 3. Paired source measurements on alpha counter. Air-filled, parallel-plate ionization chamber with linear pulse amplifier and Offner scaler.

No.	R _A (c/m)	R _B (c/m)	R _C (c/m)	b	τ ₁ (min)	δ (c/m)	ε (%)
I	294.4	313.4	603.4	2.0	13 x 10 ⁻⁶	+0.6	+0.10
II	546.5	538.2	1076.6	2.0	11 x 10 ⁻⁶	+0.4	+0.04
III	736	725	1450	2	9 x 10 ⁻⁶	-2	-0.14
IV	1323	1143	2436	2	9.7 x 10 ⁻⁶	-18	-0.74
V	2459	2514	4782	2	17.9 x 10 ⁻⁶	+13	+0.27
VI	4454	4327	8131	2	23.5 x 10 ⁻⁶	+1	+0.01
VII	5924	6245	10860	2	29.8 x 10 ⁻⁶	-7	-0.06

Result of computation: $\tau = 8.20 \times 10^{-6}$ min, $\nu = 1.147 \times 10^{-9}$ min², and $N = R + \tau R^2 + \nu R^3$

Table 4. Paired source measurements on alpha counter. Nitrogen-filled, parallel-plate ionization chamber with high-frequency amplifier and Offner scaler.*

No.	R _A (c/m)	R _B (c/m)	R _C (c/m)	b	τ ₁ (min)	δ (c/m)	ε (%)
I	41,287	41,111	81,094	2	4.10 x 10 ⁻⁷	+70	+0.09
II	80,308	70,495	145,492	3	5.44 x 10 ⁻⁷	-135	-0.09
III	108,794	123,160	216,621	2	7.73 x 10 ⁻⁷	+74	+0.03

Result of computation (see Table 6): $\tau = 1.814 \times 10^{-7}$ min, $\nu = 1.662 \times 10^{-12}$ min², and $N = R + \tau R^2 + \nu R^3$

*The losses in this instrument occur chiefly in the scaler, not in the chamber or amplifier.

Table 5. Paired source measurements on Geiger-Mueller counter.⁴ Hydrogen-filled, thin-walled glass tube (Eck and Krebs), with Neher-Harper quenching circuit and scaler. Department of Chemistry, University of Wisconsin, June 1942.

No.	R _A (c/m)	R _B (c/m)	R _C (c/m)	b	τ ₁ (min)	δ (c/m)	ε (%)
I	579	647	201	16	13 x 10 ⁻⁶	+1	+0.06
II	2,054	2,491	4,386	18	11.0 x 10 ⁻⁶	+39	+0.88
III	5,560	7,090	11,960	20	10.8 x 10 ⁻⁶	+12	+0.10
IV	7,320	9,970	16,120	20	10.6 x 10 ⁻⁶	+66	+0.41
V	17,880	18,460	33,100	20	7.4 x 10 ⁻⁶	-470	-1.42
VI	28,800	32,600	53,700	-	6.3 x 10 ⁻⁶	+630	+1.18
VII	53,500	58,200	93,200	-	7.6 x 10 ⁻⁶	-430	-0.47
VIII	65,700	77,400	114,000	-	10.8 x 10 ⁻⁶	+200	+0.18

Result of computation: $\tau = 1.217 \times 10^{-5}$ min, $\nu = -8.76 \times 10^{-11}$ min², $\phi = 5.29 \times 10^{-18}$ min³, and $N = R + \tau R^2 + \nu R^3 + \phi R^4$.

The choice of counting rates for the various sets of paired source measurements can be made from the following considerations. The R_C values should be fairly evenly distributed over the useful range of the instrument. The maximum useful rate of a counter for general purposes may be considered to be that for which the coincidence loss is about 10 or 20 per cent. Some Geiger-Mueller counters, however, have lower useful limits because high rates may permanently alter the tube characteristics. Since sets with low values of R_C require long counting intervals and enter into the computations with less weight than the higher sets, it is usually not profitable to use R_C values lower than that counting rate for which the coincidence correction is about 1 or 2%. If more than 3 or 4 sets are taken, duplication is permissible and has the advantage of indicating the consistency and reproducibility of the data.

Sample Data

To illustrate the application of the method to actual counting instruments, several series of data taken with various types of instruments are presented in Tables 2, 3, 4, and 5 together with the results of the computations. The examples have been chosen to illustrate the one-, two-, and three-parameter cases.

Preliminary Examination of Data

In order to obtain an idea of the consistency of the data and to determine the number of parameters which should be computed, a preliminary examination of the data should be made by applying equation 9 to each set of measurements. The value of τ so obtained is designated as τ_1 , since it implies a one-parameter relationship, equation 4. The values of τ_1 are included in Tables 2 to 5.

If, as in Table 2, the variations in τ_1 show no trend with counting rate, and are no greater than expected from statistical fluctuations, a one-parameter computation should suffice. The data of Tables 3 and 4 indicate a progressive increase in τ_1 (the initial decrease in Table 3 is not significant), indicating that two parameters should be computed. The data of Table 5 show a significant decrease in τ_1 followed by a significant increase, suggesting that three parameters are probably required to express the behavior of the counter.

Sample Computation

To demonstrate the manner in which the computations are set up and carried out, a typical two-parameter computation is shown in Table 6. The computations involving a different number of parameters follow similar patterns, except that with three or more parameters the solution of the simultaneous equations must be handled differently.

In the computation, sufficient significant figures should be carried to allow for the loss which occurs in solving the simultaneous equations. In the three-parameter computation of Table 5, three significant figures were lost in the solution of the equations, so eight figures were carried up to this point in order to give satisfactory accuracy in the result. Because of its length, this computation was consequently quite tedious.

Checking the Computations

The computations should be checked against the data by computing the residual (δ) and the fractional residual ($\epsilon = \delta/R_C$) for each set. If the values of ϵ are all small, that is, of the order of magnitude of P in equation 17 and are distributed randomly in sign, it is assumed that the equation obtained satisfactorily represents the behavior of the instrument. If, however, the values of ϵ are all of the same sign, or show a significant trend, an error in the computations is indicated, or an additional parameter must be computed.

In Tables 2, 3, 4, and 5, there are given for each set of measurements the values of δ and ϵ computed from the result of the least-squares computation. In each case, these quantities are small and of random sign, indicating that the equation obtained is adequate for the correction of measurements made with the instrument.

Table 6. Sample computation, illustrating method of laying out work for determining two parameters with aid of calculating machine. Data from Table 4.

Set No.	I	II	III
R_A	41,287	80,308	108,794
R_B	41,111	70,495	123,160
R_C	81,094	145,492	216,621
b	2	3	2
$R_A + R_B - R_C - b = D$	1,302	5,308	15,331
R_A^2	1.70461×10^9	6.44937×10^9	1.18361×10^{10}
R_B^2	1.69011×10^9	4.96954×10^9	1.51684×10^{10}
R_C^2	6.57623×10^9	2.11679×10^{10}	4.69246×10^{10}
$R_A^2 + R_B^2 - R_C^2 = E$	-3.18151×10^9	-9.74901×10^9	-1.99201×10^{10}
R_A^3	7.03782×10^{13}	5.17936×10^{14}	1.28770×10^{15}
R_B^3	6.94821×10^{13}	3.50328×10^{14}	1.86814×10^{15}
R_C^3	5.33293×10^{14}	3.07976×10^{15}	1.01649×10^{16}
$R_A^3 + R_B^3 - R_C^3 = F$	-3.93432×10^{14}	-2.21150×10^{15}	-7.00903×10^{15}
$DE/R_C^2 = V$	-6.29960×10^2	-2.44463×10^3	-6.50821×10^3
$DF/R_C^2 = W$	-7.78970×10^7	-5.54549×10^8	-2.28996×10^9
$EF/R_C^2 = X$	1.90338×10^{14}	1.01852×10^{15}	2.97542×10^{15}
$E^2/R_C^2 = Y$	1.53918×10^9	4.48996×10^9	8.45636×10^9
$F^2/R_C^2 = Z$	2.35376×10^{19}	2.31044×10^{20}	1.04692×10^{21}

Calculation of data in Table 6:

$\Sigma V = H$	-9.58280×10^3	HK	-4.00971×10^{19}
$\Sigma W = J$	-2.92240×10^9	JL	-4.23324×10^{19}
$\Sigma X = K$	4.18428×10^{15}	$HK - JL$	$+2.2353 \times 10^{18}$
$\Sigma Y = L$	1.44855×10^{10}	LM	$+1.88529 \times 10^{31}$
$\Sigma Z = M$	1.30150×10^{21}	K^2	$+1.75082 \times 10^{31}$
		$LM - K^2$	$+1.3447 \times 10^{30}$
JK	-1.2281×10^{25}	$\frac{JK - HM}{LM - K^2} = \tau$	$+1.814 \times 10^{-7}$
HM	-1.24720×10^{25}		
$JK - HM$	$+2.439 \times 10^{23}$	$\frac{HK - JL}{LM - K^2} = \nu$	$+1.662 \times 10^{-12}$

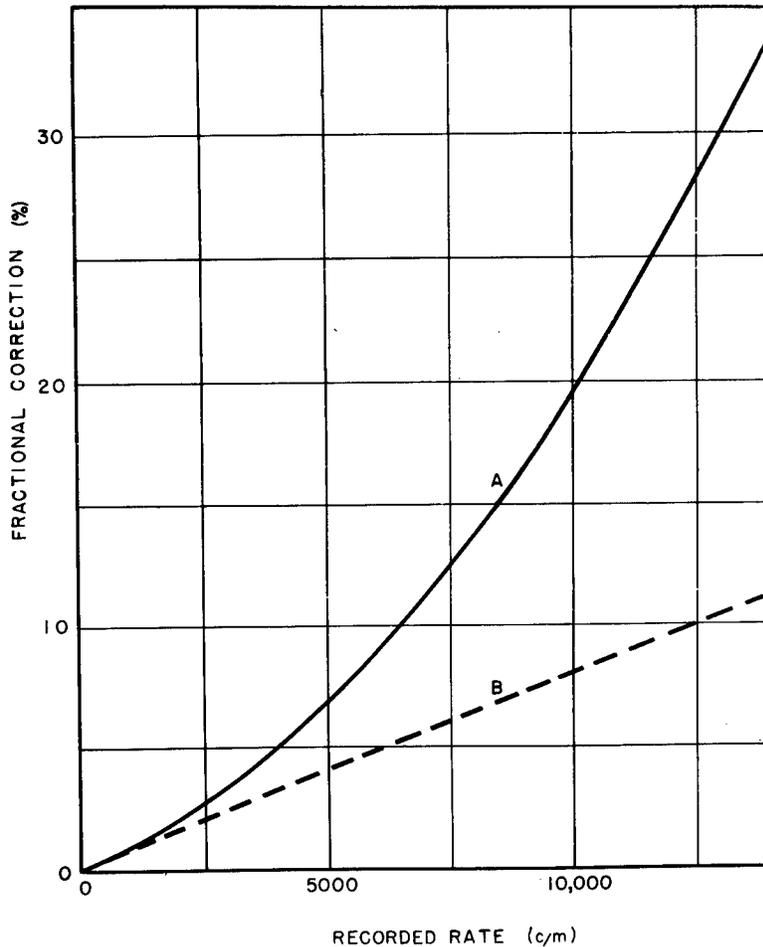


Figure 1. Coincidence corrections for alpha counter. Data of Table 3.

A: Found by present method. B: Previously assumed for this type of instrument.

Graphical Illustration of Results

For two of the examples considered, the fractional correction has been plotted as a function of the recorded rate.

Figure 1 shows the behavior of the alpha counter from which the data of Table 3 were taken. For comparison, the dash line indicates the previously assumed losses of such instruments, 0.8% per 1000 counts/minute.¹³

The difference between the two curves indicates the necessity of determining coincidence corrections separately for every instrument.

In Figure 2 the continuous line shows the results of the three-parameter computation based on the Geiger-Mueller counter data of Table 5. The dash line is the result of the independent analysis of the same data by the method of successive graphical approximations.² The two curves coincide within the accuracy of the measurements, indicating the validity of both methods. The dash curve

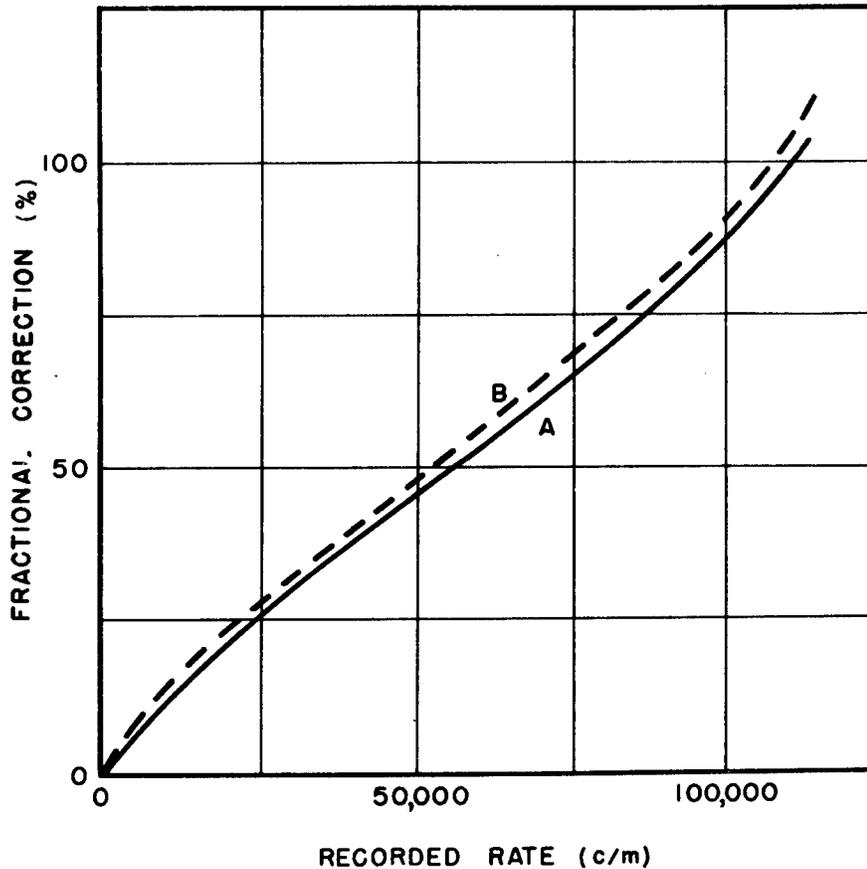


Figure 2. Coincidence corrections for GM counter. Data of Table 5.

A: Found by present method. B: Found by method of successive graphical approximations.

is perhaps the better of the two since it yields a value for $\Sigma \epsilon^2$ of 0.00005, whereas that obtained by the present method is 0.00046. A fourth parameter would presumably reduce the difference between the two curves.

To facilitate the correction of counter data, it is often convenient to use a graph showing the correction ($N-R$) as a function of the recorded rate R , instead of applying the correction equation to the individual results. The plot may be made on Cartesian or logarithmic coordinate paper, depending on the range of values. Such plots are shown in Figures 3 and 4.

ACKNOWLEDGMENTS

The data of Tables 2 and 3 were obtained through the cooperation of Robert Jolly. The computations were made by Robert R. Jones, Miss Virginia Webb, and Mrs. Gertrude Paas, through the cooperation of Benjamin Butler.

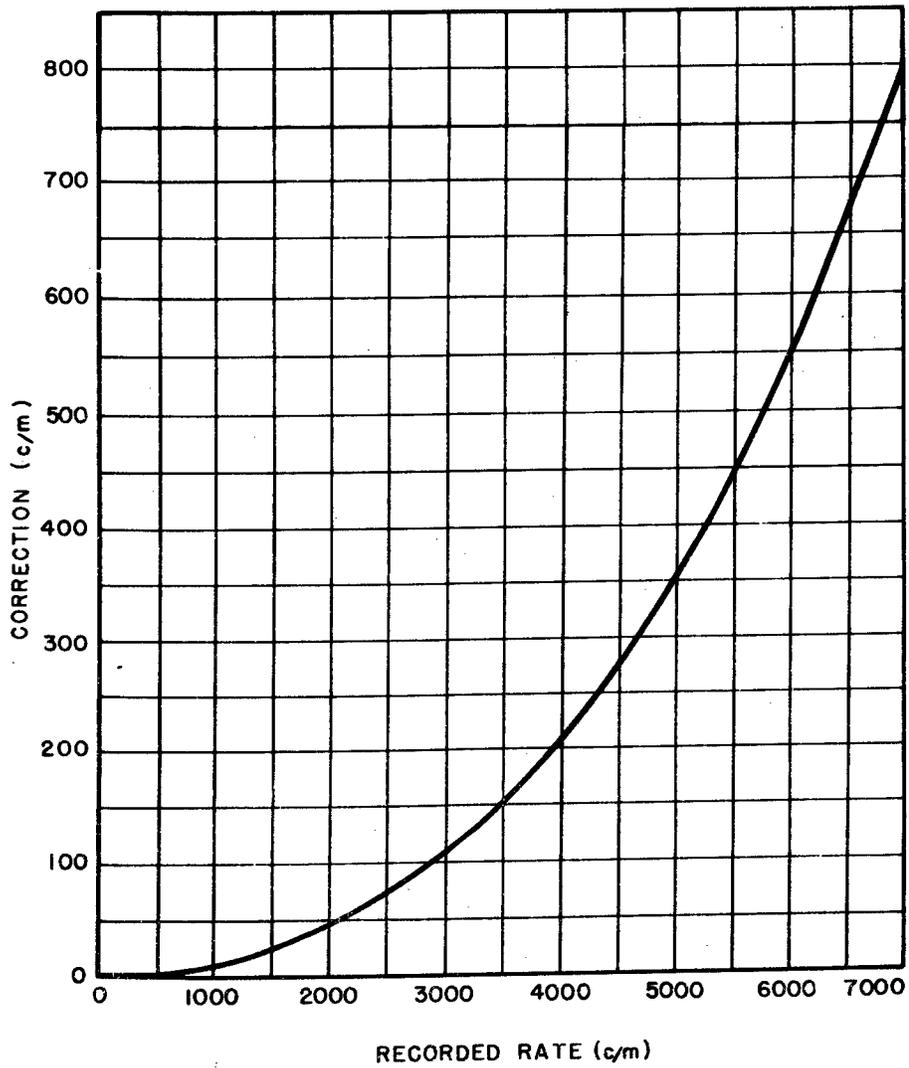


Figure 3. Coincidence correction chart for alpha counter, linear plot. Data of Table 3.

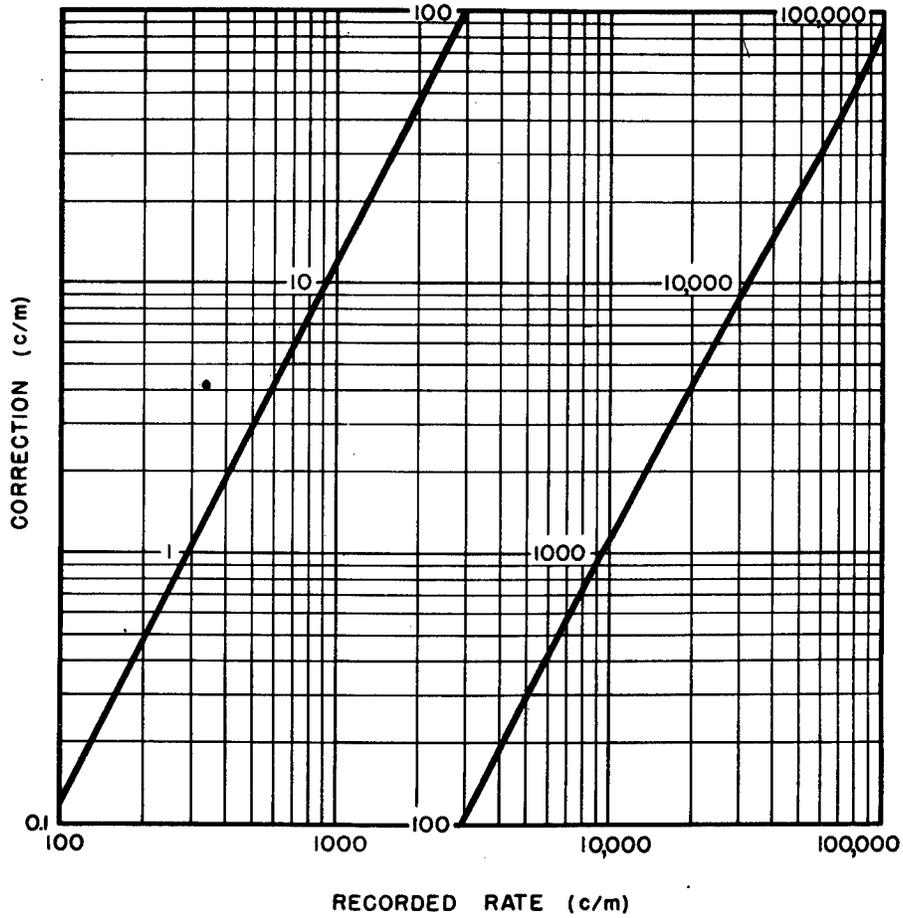


Figure 4. Coincidence correction chart for GM counter, logarithmic plot. Data of Table 5.

REFERENCES

1. Beers, Rev. Sci. Instruments 13:72 (1942).
2. Kohman, Ph.D. thesis, University of Wisconsin, 50-70; abstract in Phys. Rev. 65:63 (1944).
3. Moon, J. Sci. Instruments 14:189 (1937).
4. Hull, Rev. Sci. Instruments 11:404 (1940).
5. Hull and Seelig, Phys. Rev. 60:553 (1941).
6. Brady, Crawford, Ghiorso, Jaffey, Jarrett, Kohman, and Scott, unpublished work at Metallurgical Laboratory, University of Chicago, and Clinton National Laboratory.
7. Metcalf and Hennessee, Metallurgical Project report CP-2582 (1945).
8. Kohman, Metallurgical Laboratory memorandum N-1487 (1944).
9. Johnson and Street, J. Franklin Inst. 215:239 (1939).
10. Ruark and Brammer, Phys. Rev. 52:322 (1937).
11. Volz, Z, Physik 93:539 (1935).
12. Schiff, Phys. Rev. 50:88 (1936).
13. Jaffey, Kohman, and Crawford, A Manual on the Measurement of Radioactivity, Metallurgical Project report CC-1602 (1944).
14. Danforth, Phys. Rev. 46:1026 (1934).